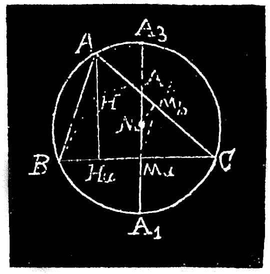
Solution by F. M. McGAW, A. M., Professor of Mathematics, Bordentown Military Institute, Bordentown, New Jersey, and the PROPOSER.

Let $M_aA_1=M_aA_2$, $A_2A_3=AH$, to prove A_3 on the circumference of the circle. Since A_2A_3 is a line through M, the center of the circle, the proposition is in effect to prove A_3 one extremity of the diameter through M_a.



By the conditions $AH=A_2A_3$, and is parallel to it, therefore AHA_3A_2 is a parallelogram.

Also triangles BHA and M_aMM_b are similar, hence since $2M_aM_b=AB$, we have $AH=2MM_a$.

Therefore, $A_1A_3=A_2A_3+A_2M_a+M_aA_1$
$=AH+2M_aA_1$
$=2M_aM+2M_aA_1$
$=2(MA_1)=2r$, hence A_3 is extremity of diameter.

Q. E. D.

Also solved by *CHAS. C. CROSS*, and *J. W. SCROGGS*.
Mr. Cross furnished two different solutions.

72. Proposed by O. W. ANTHONY, M. Sc., Professor of Mathematics, Columbian University, Washington, D. C.

If a line with its extremities upon two curves move in any manner whatever, (the line may vary in length), and P a point upon the line which divides it in the ratio $m:n$ describe a curve, the area of this curve will be given by the formula—

$$A=\frac{(m^2+nm)A_1+(n^2+mn)A_2-mnA_3}{(m+n)^2}.$$

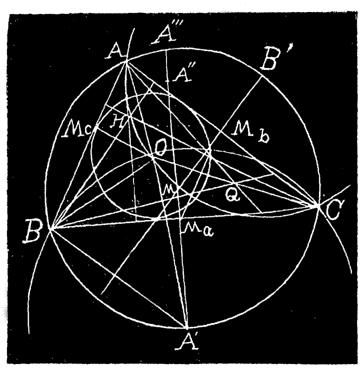
No solution of this problem has been received.

73. Proposed by ROBERT J. ALEY, A. M., Ph. D., Professor of Mathematics, Indiana University, Bloomington, Indiana.

Prove by pure geometry: (1) A', B', and C' are the middle points of the arcs BC, CA, and AB respectively. With these points as centers, circles are described passing through B and C, C and A, and A and B respectively. Prove that these circles intersect in O, the center of the incircle of the triangle ABC; (2) that O, the center of the incircle, is Nagel's point of the triangle formed by joining the middle points of the sides.

Solution by CHARLES C. CROSS, Laytonsville, Maryland, and the PROPOSER.

(1) AO cuts the circumcircle at A', for AO bisects angle A and also its subtending arc. $\measuredangle OBA'=\frac{1}{2}(A+B)$.



$\measuredangle BOA'=\frac{1}{2}(A+B)$ for it is exterior angle to triangle BOA.

$\therefore$ triangle $A'BO$ is isosceles.

$A'B=A'O$. By similar reasoning it is proved that $B'A=B'O$ and $C'A=C'O$.

$\therefore$ The circles intersect in O.

(2) It is a well known property of Nagel's point that AQ and OM_a, BQ and OM_b, CQ and OM_c are respectively parallel.

The triangle $M_aM_bM_c$ is similar to the triangle ABC.

$$\measuredangle OM_aM_c = \measuredangle QAC.$$
$$\measuredangle OM_bM_c = \measuredangle QBC.$$
$$\measuredangle OM_cM_a = \measuredangle QCA.$$

$\therefore$ O with respect to the triangle $M_aM_bM_c$, is located precisely as Q is with respect to the triangle ABC.

Hence O is Nagel's point of triangle $M_aM_bM_c$.

Also solved by *F. M. McGAW* and *G. B. M. ZERR*.

74. Proposed by ROBERT J. ALEY, A. M., Ph. D., Professor of Mathematics, Indiana University, Bloomington, Indiana.

Let O be the center of the inscribed circle. AO produced meets the circumcircle in A'. Find the ratio of AO to OA'.

I. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics, Ohio University, Athens, Ohio.

The coördinates of A are $\left(\frac{2\triangle}{a},\ 0,\ 0\right)$; of O, $(r,\ r,\ r)$; and of A', those of the intersection of $\beta-\gamma=0\ldots\ldots(1)$, with $a\beta\gamma+b\alpha\gamma+c\alpha\beta=0\ldots\ldots(2)$, having the constant relation $a\alpha+b\beta+c\gamma=2\triangle\ldots\ldots(3)$. These give for the coördinates

of A' $\left(-\frac{(b+c)^2}{a^2}-\frac{2\triangle}{a},\quad \frac{(b+c)^3}{a^3}+\frac{2\triangle(b+c)}{a^2},\quad \frac{(b+c)^3}{a^3}+\frac{2\triangle(b+c)}{a^2}\right)$.

The distance d between $(\alpha_1,\ \beta_1,\ \gamma_1)$ and $(\alpha_2,\ \beta_2,\ \gamma_2)$ is given by

$$d^2=-\frac{abc}{4\triangle^2}\{a(\beta_1-\beta_2)(\gamma_1-\gamma_2)+b(\gamma_1-\gamma_2)(\alpha_1-\alpha_2)+c(\alpha_1-\alpha_2)(\beta_1-\beta_2)\}\quad\ldots(4).$$

Putting $\alpha_1=(2\triangle/a)$, $\beta_1=C$, $\gamma_1=0$; $\alpha_2=\beta_2=\gamma_2=r$,

$$\overline{AO}^2=bcr(b+c-a)/2\triangle\quad\ldots\ldots(5).$$

Putting α_1, β_1, γ_1 equal respectively to the coördinates of A', and $\alpha_2=\beta_2=\gamma_2=r$ as before, in (4), we get an expression for $\overline{OA'}^2$.

We can then express the ratio of OA to OA'.

II. Solution by J. SCHEFFER, A. M., Hagerstown, Maryland.

The point A' is evidently the middle point of arc BC. Since $\angle A'OC=\frac{1}{2}(A+C)$ and $\angle A'CO=\frac{1}{2}(A+B)$, $OA'=A'C=A'B$.

From Ptolemy's theorem, $ACA'B$ being a cyclic quadrilateral,

$$AB\times A'C+AC\times A'B=AA'\times BC, \text{ or}$$
$$c\times OA'+b\times OA'=(AO+OA')a.$$
$$\therefore OA : OA'=b+c-a : a=3-a : 2a.$$

Also solved by *G. B. M. ZERR* and *CHAS. C. CROSS*.